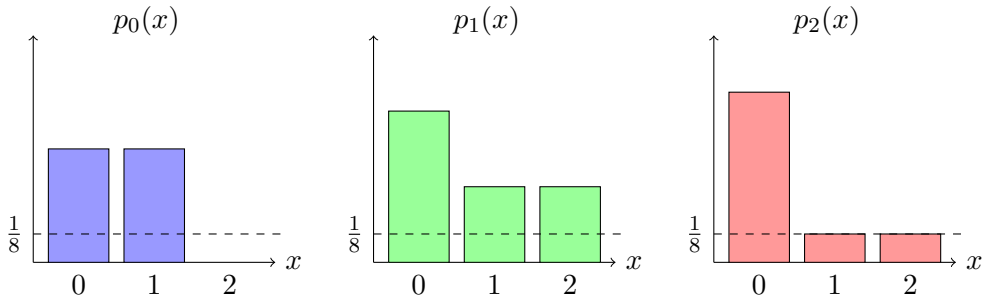
Quiz 7: Quantum Compression and Entropy

Question 1. Consider the probability distributions.



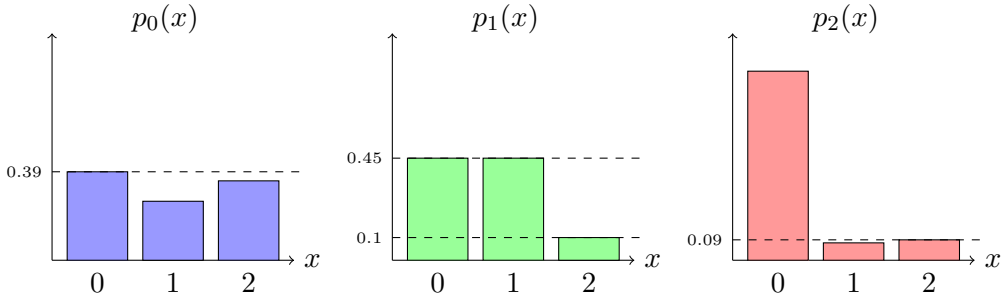
Which of these has the smallest optimal zero-error compression $C^0(p_i)$?

(a) p_0, (b) p_1, (c) p_2,

Question 2. Which of the probability distributions from the previous question has the smallest optimal compression with error 1/4, that is, $C^{1/4}(p_i)$?

(a) p_0, (b) p_1, (c) p_2,

Question 3. Consider the following probability distributions.



Which one has the largest Shannon entropy $H(p_i)$?

(a) p_0, (b) p_1, (c) p_2,

Question 4. Let p_X be a probability distribution. Which of the following statements about the typical sets $T_{n,\varepsilon}(p_X)$ are true in general?

(a) $T_{n,\varepsilon}(p_X)$ contains the most likely outcomes for the variables $\mathbf{X}^n$.

(b) $T_{n,\varepsilon}(p_X)$ gets larger as $n \to \infty$.

(c) The probability that a sample of $\mathbf{X}^n$ is in $T_{n,\varepsilon(p_X)}$ is equal to one.

Question 5. Suppose Alice wants to send Bob a message consisting of n bits. Each bit is independently 0 with probability 3/4, and 1 with probability 1/4. How many bits does she need to send in the limit of large n?

$$\text{(a) } \frac{n}{4}(6 - 3\log 3), \quad \text{(b) } n\left(2 - \frac{3}{4}\log 3\right), \quad \text{(c) } 2n, \quad \text{(d) } n,$$