Quiz 4: Quantum Channels

Question 1. Which of the following statements are true?

(a) Any map of the form $\rho \mapsto A\rho A^\dagger$ is completely positive.

(b) The transpose map $\rho \mapsto \rho^T$ is completely positive.

(c) If $\Phi_{A\to B}$ is completely positive, the so is $\Phi_{A\to B} \otimes \mathcal{I}_{C\to C}$.

(d) The composition of two completely positive maps is also completely positive.

Question 2. Which of the following statements are true?

(a) Any map of the form $\rho \mapsto A\rho A^\dagger$ is trace preserving.

(b) The Hermitian conjugate map $M \mapsto M^\dagger$ is trace preserving.

(c) The tensor product of two trace preserving maps is also trace preserving.

(d) The composition of two trace preserving maps is also trace preserving.

Question 3. Which of the following are quantum channels?

(a) $\Phi_{A\to B}\colon \rho_A \mapsto \sigma_B \operatorname{tr}[\rho_A]$, for some fixed $\rho_B \in S(B)$.

(b) $\Phi_{A\to\mathbb{C}^2}\colon \rho \mapsto (|0\rangle\langle 0| \operatorname{tr}[\rho^2] + |1\rangle\langle 1|(1-\operatorname{tr}[\rho^2]))\operatorname{tr}[\rho]$.

(c) $\Phi_{\mathbb{C}^2\to\mathbb{C}}\colon \rho \mapsto \operatorname{tr}[\rho Z]$.

(d) $\Phi_{A\to A}\colon \rho \mapsto 2\cdot \frac{1}{\dim(\mathcal{H}_A)}\mathbb{1}_A - \rho$.

Question 4. Consider the replacement channel $\Phi_{A\to B}\colon \rho_A \mapsto \sigma_B \operatorname{tr}[\rho_A]$. Which of the following statements are true?

(a) $\Phi_{A\to B}$ has a Kraus representation given by $X_a = \sqrt{\sigma_B}\langle a|$, where $\{|a\rangle\}_a$ is a basis for A.

(b) If $\sigma_B = |\psi\rangle\langle\psi|$ is a pure state, $\Phi_{A\to B}$ has a Kraus representation given by $X_a = |\psi\rangle\langle a|$, where $\{|a\rangle\}_a$ is a basis for A.

Question 5. Suppose that we have a machine that takes one qubit as its input. With probability 1/2, the machine will apply the Pauli operator X (a bit flip), and otherwise the machine will do nothing to the state. What is the quantum channel corresponding to the machine?

(a) $\rho \mapsto \frac{1}{2}\mathbb{1}\operatorname{tr}[\rho]$.

(b) $\rho \mapsto \frac{1}{2}(\rho + X\rho X)$.

(c) $\rho \mapsto \frac{1}{2}(\rho + \frac{1}{2}\mathbb{1}\operatorname{tr}[\rho])$.

(d) $\rho \mapsto \frac{1}{2}(X\rho X + \frac{1}{2}\mathbb{1}\operatorname{tr}[\rho])$.

Question 6. Let Φ be the channel from the previous question. What is the action of Φ on the Bloch sphere?

(a) Reflection in the y, z-plane.

(b) Projection onto the y, z-plane.

(c) A rotation of π radians about the x-axis.

(d) Projection onto the x-axis.

Question 7. Suppose that we have a machine taking a one quibit as its input The machine measures in the Z-basis, i.e. $\{|0\rangle, |1\rangle\}$. If the measurement outcome is 0, then the machine prepares the state $|\psi\rangle$, else it prepares the state $|\phi\rangle$. What is the quantum channel corresponding to the machine?

(a) $\rho \mapsto \langle 0|\rho|0\rangle |\psi\rangle\langle\psi| + \langle 1|\rho|1\rangle |\phi\rangle\langle\phi|$.

(b) $\rho \mapsto \langle\psi|\rho|\psi\rangle |0\rangle\langle 0| + \langle\phi|\rho|\phi\rangle |1\rangle\langle 1|$.

(c) $\rho \mapsto \frac{1}{2}(\langle 0|\rho|0\rangle |\psi\rangle\langle\psi| + \langle 1|\rho|1\rangle |\phi\rangle\langle\phi|)$.

(d) $\rho \mapsto \frac{1}{2}(\langle\psi|\rho|\psi\rangle |0\rangle\langle 0| + \langle\phi|\rho|\phi\rangle |1\rangle\langle 1|)$.