Quiz 1: Quantum States

Question 1. Which of the following are *not* density matrices?

$$A = \frac{1}{4}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \in \mathbb{C}^{2\times 2}, \qquad\qquad B = \frac{1}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \in \mathbb{C}^{2\times 2},$$

$$C = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \in \mathbb{C}^{4\times 4}, \qquad\qquad D = \begin{pmatrix} 1 \end{pmatrix} \in \mathbb{C}^{1\times 1}.$$

Question 2. Which of the following density matrices correspond to *pure* states?

$$A = \frac{1}{4}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \in \mathbb{C}^{2\times 2}, \qquad\qquad B = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \in \mathbb{C}^{2\times 2},$$

$$C = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \in \mathbb{C}^{4\times 4}, \qquad\qquad D = \begin{pmatrix} 1 \end{pmatrix} \in \mathbb{C}^{1\times 1}.$$

Question 3. Which of the following density matrices correspond to *classical* states?

$$A = \frac{1}{4}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \in \mathbb{C}^{2\times 2}, \qquad\qquad B = \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \in \mathbb{C}^{2\times 2},$$

$$C = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \in \mathbb{C}^{4\times 4}, \qquad\qquad D = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \mathbb{C}^{4\times 4}.$$

Question 4. Recall that $S(\mathcal{H})$ denotes the set of density matrices on $\mathcal{H}$. Consider the following statements:

(I) $\rho_1 \in S(\mathcal{H})$ and $\rho_2 \in S(\mathcal{H})$.
(II) $\rho_1\rho_2 \in S(\mathcal{H})$.

Which of the following holds?

(A) (I) $\implies$ (II).
(B) (I) $\impliedby$ (II).
(C) (I) $\iff$ (II).
(D) None of those.