Problem sheet 6: Distance Measures

Problem 1 (HW). **Typical subspaces.** Prove the following from the lecture: Let $\rho_A \in S(A)$ and $\varepsilon > 0$.

(a) The non-zero eigenvalues of $\Pi_{n,\varepsilon}\rho_A^{\otimes n} = \Pi_{n,\varepsilon}\rho_A^{\otimes n}\Pi_{n,\varepsilon}$ are all in the interval

$$[2^{-n(H(\rho_A)+\varepsilon)}, 2^{-n(H(\rho_A)-\varepsilon)}].$$

(b) The dimension of the typical subspace is bounded by

$$\dim(S_{n,\varepsilon}(\rho_A)) \leq 2^{n(H(\rho_A)+\varepsilon)}.$$

(c) We have

$$\lim_{n\to\infty} \operatorname{tr}[\Pi_{n,\varepsilon}\rho_A^{\otimes n}] = 1,$$

that is, as $n \to \infty$, if we measure whether we are in the typical subspace (corresponding to the measurement $\{\Pi_{n,\varepsilon}, \mathbb{1}_{A^n} - \Pi_{n,\varepsilon}\}$), the probability of being in the typical subspace goes to 1.