

## Problem sheet 4: Quantum Channels

**Problem 1** (HW). For the following scenarios, write down the quantum channel that models them.

- (a) We have a qubit system. With probability  $1/2$  we do nothing and with probability  $1/2$  we discard the state and prepare the state  $|0\rangle$ .
- (b) We have two qubit systems  $A$  and  $B$ . We replace the state on the  $B$ -system with  $|0\rangle$  and apply a Pauli  $Z$  operator to the  $A$ -system.
- (c) We start with system  $A$ , add system  $E$  in state  $|0\rangle$ . We apply a global unitary  $U$  on  $A$  and  $E$  and then discard the system  $E$ .

**Problem 2** (HW). Prove that the superoperators defined at lecture are in fact quantum channels.

- (a) Depolarizing channel. *Hint: compute the Choi operator.*
- (b) Dephasing channel. *Hint: find a Kraus representation.*
- (c) Erasure channel.
- (d) Replacement channel. *Hint: Prepare a purification of  $\rho$  and discard both the input system  $A$  and the purifying system. This gives a Stinespring representation of the replacement channel.*