

# Homework for Quantum Information

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## 1 Orthogonal bases

**Exercise 1.1.** Compute the vector  $|u\rangle = (2 + i, 1 - i)$  with respect to the orthonormal basis

$$\left( \frac{1}{\sqrt{2}}(1, i), \frac{1}{\sqrt{2}}(i, 1) \right).$$

**Exercise 1.2.** Compute the vector  $|u\rangle = (1 + i, 2 + i)$  with respect to the orthonormal basis

$$((0, -i), (i, 0)).$$

**Exercise 1.3.** Compute the vector  $|u\rangle = (2 - i, i)$  with respect to the orthonormal basis

$$\left( \frac{1}{\sqrt{2}}(1, 1), \frac{1}{\sqrt{2}}(1, -1) \right).$$

**Exercise 1.4.** Compute the vector  $|u\rangle = (1 + i, i)$  with respect to the orthonormal basis

$$\left( \frac{1}{\sqrt{2}}(-i, i), \frac{1}{\sqrt{2}}(i, i) \right).$$

## 2 Tensor products

**Exercise 2.1.** Write out  $Z \otimes Z$ .

**Exercise 2.2.** Write out  $X \otimes H$ .

**Exercise 2.3.** Write out  $H \otimes Y$ .

**Exercise 2.4.** Write out  $\frac{1}{\sqrt{2}}(Y + Z) \otimes Y$ .

### 3 Measurements

**Exercise 3.1.** Compute the probability that measuring the observable  $H \otimes H$  on the state  $(1 + i, i) \otimes (1 - i, 2i)$  yields the value 1.

**Exercise 3.2.** Compute the probability that measuring the observable  $X$  on the state  $(1+i, 1-i)$  yields the value 1.

**Exercise 3.3.** Compute the probability that measuring the observable  $H$  on the state  $(1 - i, 2)$  yields the value 1.

**Exercise 3.4.** Compute the probability that measuring the observable  $H \otimes Z$  on the state  $(1 - i, 2) \otimes (1 + i, i)$  yields the value 1.

**Exercise 3.5.** Compute the expectation value of the observable  $X$  on the state  $|\varphi\rangle = (2 - i, i)$ .

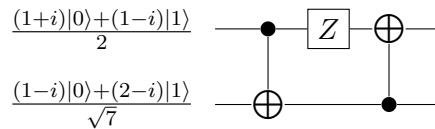
**Exercise 3.6.** Compute the expectation value of the observable  $H \otimes \frac{1}{\sqrt{2}}(Y + Z)$  on the state  $(2 - i, i) \otimes (1 - i, 2)$ .

**Exercise 3.7.** Compute the expectation value of the observable  $\frac{1}{\sqrt{2}}(Y + Z) \otimes Z$  on the state  $(1 + i, 1 - i) \otimes (1 - i, 2 - i)$ .

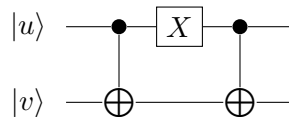
**Exercise 3.8.** Compute the expectation value of the observable  $H$  on the state  $(2 - i, i)$ .

### 4 Circuits

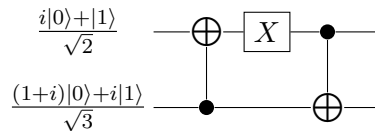
**Exercise 4.1.** Compute the output of the following circuit:



**Exercise 4.2.** Compute the output of the following circuit for some vectors  $|u\rangle$  and  $|v\rangle$ :



**Exercise 4.3.** Compute the output of the following circuit:



**Exercise 4.4.** Compute the output of the following circuit:

