

Exercises for Quantum Information

Sheet 4 — Discrete Fourier Transform

0. For $g, h \in \mathbb{Z}_{r_1} \times \dots \times \mathbb{Z}_{r_m}$ show that $\chi^{-1}(g) = \chi(g)^*$ and $\chi_g(h) = \chi_h(g)$.
1. Show that DFT and DFT^{-1} are complex conjugate.
2. Describe the canonical basis and the basis of characters of the space of functions from G to \mathbb{C} , where:
 - $G = \mathbb{Z}_2 \times \mathbb{Z}_3$,
 - $G = \mathbb{Z}_9$,
 - $G = \mathbb{Z}_3 \times \mathbb{Z}_3$.

Compute DFT and IFT matrices for all the cases described above.

3. Are there some special relations between DFT and IFT matrices for \mathbb{Z}_2^m ?
4. Construct a circuit computing IFT for \mathbb{Z}_8 explicitly.