

Mathematical Analysis 2

Metric spaces and continuity: mock exam

Time: 45 minutes Total: 10 points

Name: _____

Score: _____ / 10

This practice paper illustrates the format and level of the test. Try it in 45 minutes under the same conditions as the test.

Written or printed materials are allowed. Electronic devices are not allowed. Work independently and write your solutions on separate paper. Justify all answers. You may cite results from the course, but explain why their hypotheses apply. Unless another metric is specified, use the Euclidean metric.

1. A weighted maximum metric

[5 points]

For $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in $\mathbb{R}^2$, define

$$\sigma(x, y) = \max\{2|x_1 - y_1|, |x_2 - y_2|\}.$$

- (a) Prove that σ is a metric on $\mathbb{R}^2$. [3 points]
- (b) Describe and sketch the open ball $B_\sigma((-1, 1), 2)$. Give the vertices of its boundary and state whether the boundary is included. [2 points]

2. A function of two variables

[5 points]

Consider the real-valued function

$$G(x, y) = \sqrt{\ln(x^2 + 4y^2)}.$$

- (a) Find its domain D . Decide separately whether D is open and whether D is closed in $\mathbb{R}^2$, and justify both answers. [3 points]
- (b) Determine where $G : D \rightarrow \mathbb{R}$ is continuous, including at boundary points of D . [2 points]